Statistics Tutorial: Poisson Distribution

Attributes of a Poisson Experiment

A **Poisson experiment** is a statistical experiment that has the following properties:

- The experiment results in outcomes that can be classified as successes or failures.
- The average number of successes (μ) that occurs in a specified region is known.
- The probability that a success will occur is proportional to the size of the region.
- The probability that a success will occur in an extremely small region is virtually zero.

Note that the specified region could take many forms. For instance, it could be a length, an area, a volume, a period of time, etc.

Notation

The following notation is helpful, when we talk about the Poisson distribution.

- *e*: A constant equal to approximately 2.71828. (Actually, *e* is the base of the natural logarithm system.)
- μ: The mean number of successes that occur in a specified region.
- *x*: The actual number of successes that occur in a specified region.
- P(*x*; μ): The **Poisson probability** that exactly *x* successes occur in a Poisson experiment, when the mean number of successes is μ.

Poisson Distribution

A **Poisson random variable** is the number of successes that result from a Poisson experiment. Theprobability distribution of a Poisson random variable is called a **Poisson distribution**.

Given the mean number of successes (μ) that occur in a specified region, we can compute the Poisson probability based on the following formula:

Poisson Formula. Suppose we conduct a Poisson experiment, in which the average number of successes within a given region is μ. Then, the Poisson probability is:

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!},$$

where *x* is the actual number of successes that result from the experiment, and *e* is approximately equal to 2.71828.

The Poisson distribution has the following properties:

- The mean of the distribution is equal to μ .
- The variance is also equal to μ .

Example 1

The average number of homes sold by the Acme Realty company is 2 homes per day. What is the probability that exactly 3 homes will be sold tomorrow?

Solution: This is a Poisson experiment in which we know the following:

- $\mu = 2$; since 2 homes are sold per day, on average.
- $x = 3$; since we want to find the likelihood that 3 homes will be sold tomorrow.
- $e = 2.71828$; since *e* is a constant equal to approximately 2.71828.

We plug these values into the Poisson formula as follows:

$P(x; \mu) = (e^{-\mu}) (\mu^x) / x!$

$P(3; 2) = (2.71828^{-2}) (2^3) / 3!$

$P(3; 2) = (0.13534) (8) / 6$

$P(3; 2) = 0.180$

Thus, the probability of selling 3 homes tomorrow is 0.180 .

Poisson Calculator

Clearly, the Poisson formula requires many time-consuming computations. The Stat Trek Poisson Calculator can do this work for you - quickly, easily, and error-free. Use the Poisson Calculator to compute Poisson probabilities and cumulative Poisson probabilities. The calculator is free. It can be found under the Stat Tables tab, which appears in the header of every Stat Trek web page.

Poisson Calculator

Cumulative Poisson Probability

A **cumulative Poisson probability** refers to the probability that the Poisson random variable is greater than some specified lower limit and less than some specified upper limit.

Example 1

Suppose the average number of lions seen on a 1-day safari is 5. What is the probability that tourists will see fewer than four lions on the next 1-day safari?

Solution: This is a Poisson experiment in which we know the following:

- $\mu = 5$; since 5 lions are seen per safari, on average.
- $x = 0, 1, 2,$ or 3; since we want to find the likelihood that tourists will see fewer than 4 lions; that is, we want the probability that they will see 0, 1, 2, or 3 lions.
- $e = 2.71828$; since *e* is a constant equal to approximately 2.71828.

To solve this problem, we need to find the probability that tourists will see 0, 1, 2, or 3 lions. Thus, we need to calculate the sum of four probabilities: $P(0; 5) + P(1; 5) + P(2; 5) + P(3; 5)$. To compute this sum, we use the Poisson formula:

$P(x \leq 3, 5) = P(0; 5) + P(1; 5) + P(2; 5) + P(3; 5)$

$P(x \leq 3, 5) = [(e^{-5})(5^0) / 0!] + [(e^{-5})(5^1) / 1!] + [(e^{-5})(5^2) / 2!] + [(e^{-5})(5^3) / 3!]$

$P(x \leq 3, 5)$ = [(0.006738)(1) / 1] + [(0.006738)(5) / 1] + [(0.006738)(25) / 2] + [(0.006738)(125) / 6]

$P(x \leq 3, 5)$ = [0.0067] + [0.03369] + [0.084224] + [0.140375]

$P(x \leq 3, 5) = 0.2650$

Thus, the probability of seeing at no more than 3 lions is 0.2650.